

VCG Mechanisms

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I VCG Auctions

I.1 Terminology

A one-item VCG auction is referred to as a Vickrey auction.

I.2 Rules

Each player submits a report of their valuation(s). The player(s) with highest valuation(s) are assigned the object(s), and are required to pay their total negative externality to the other players.

I.3 Properties

Under the following assumptions:

1. Valuations are private (i.e.: don't depend on other player's information)
2. Players cannot manipulate the auction by colluding (merging)
3. Players cannot manipulate the auction by pretending to be two or more players
4. All players' preferences over the state of the world are quasi-linear with respect to their spending on all other goods

a VCG auction has the following properties:

1. It is a weakly dominant strategy to give a truthful report
2. Hence the outcome of the auction is always Pareto efficient, although not necessarily revenue maximising for the auctioneer

2 VCG Preference Revelation Mechanisms for Public Goods Provision

2.1 Assumptions

1. Agents can be compelled to participate in the process
2. Valuations of the public good are private (i.e.: don't depend on other agent's information)
3. Agents cannot manipulate the process by colluding (merging)
4. Agents cannot manipulate the process by pretending to be two or more players
5. All agents' preferences over the state of the world are quasi-linear with respect to their spending on all other goods

2.2 Preliminaries

G	Level of public good provision. The payment scheme for each possible level of G must be fixed by the government in advance.
$r_i(G)$	Agent i 's report of how her net utility varies as a function of the level of public good provision G , given the payment scheme as fixed by the government.
G^*	$\arg \max_G \left[\sum_{i \in N} r_i(G) \right]$

2.3 Rules

Each agent i submits $r_i(G)$.

The level of the public good provided is then set at G^* .

Each agent i pays their dues according to the prior payment scheme, plus a supplementary tax

$$f_{j \neq i} - \sum_{j \neq i} r_j(G^*)$$

where $f_{j \neq i}$ is a function that does not depend on $r_i(G)$.

Supplementary taxes are sometimes referred to as 'transfers'.

Usually $f_{j \neq i} := \max_G \sum_{j \neq i} r_j(G)$. In this case, the supplementary tax is referred to as the ‘Vickrey payment’.

If $f_{j \neq i} := 0$, then the VCG mechanism is referred to as a ‘team mechanism’.

2.4 Pivotal Agents

Agent i is pivotal iff her presence changes the outcome, i.e. iff:

$$\arg \max_G \left[\sum_{j \neq i} r_j(G) \right] \neq G^*$$

2.5 Properties

1. It is a weakly dominant strategy to give a correct report
2. Hence if no agent is pivotal then the outcome must be Kaldor–Hicks efficient, although usually not – because the payment scheme must be pre-determined in advance – a Pareto improvement on the original scenario. (Things are slightly more complicated if one or more agents are pivotal, because the mechanism only works if supplementary taxes collected are not redistributed back to the agents – but the size of the distortion introduced by this complication is very small.)
3. Hence if no agent is pivotal then the outcome is socially optimal according to a Benthamite SWF.

2.6 The Equity Objection

Because the payment scheme must be fixed in advance, one can object to the VCG mechanism on the grounds that it makes some people worse off.