Oxford Game Theory: Synopsis

We can categorise the games that we'll study in this course in the following two-way typology:

		simultaneous move	sequential move
the players have:	**perfect information**	*strategic games*	*extensive-form games with perfect information*
	imperfect information	*Bayesian games*, and *strategic games with a correlating device*	*extensive-form games with imperfect information*

A rough outline of the course (in the order that it will be taught) is as follows:

1. *Strategic games*, and solution concepts (Nash equilibrium, rationalizability, etc.) that yield predictions of play for strategic games in different contexts.
2. *Correlating devices*
3. *Bayesian games*
   a. 'Harsanyi purification': One of the criticisms of the *mixed Nash equilibrium* solution concept for *strategic games* is that in a *mixed* Nash equilibrium, the players who are mixing necessarily *lack* a *strict* incentive to mix as they do (they must needs be indifferent between each of the pure strategies they mix over). They could deviate to a pure strategy without paying any penalty – and yet their doing so would destroy the equilibrium.

      Harsanyi noticed that if:

      > each player's payoff every time she plays a *strategic game* is subject to a small random perturbation that only she can observe,

      then:

      > we can understand the mixed Nash equilibrium of the *unperturbed game* as a slightly imprecise description of a *pure* Nash equilibrium of the *perturbed game*,
      >
      > in which each player has a *strict* incentive to play their chosen strategy.

      Thus, the idea of 'Harsanyi purification' enhances the plausibility of the mixed Nash equilibrium solution concept (in cases where game-payoffs are likely to be subject to small, random, and *private* perturbations).

   b. *Global games*: a particular sort of Bayesian game; one in which each player's uncertainty takes the form 'exactly *what game* am I playing here?'
4. Formal epistemology: We use some very basic formal modelling to make sense of concepts like 'common knowledge', 'self-evident', etc.
5. *Behavioural game theory*. The solution concepts that we study for the majority of the Oxford course make sense in the context of an implicit assumption that all of the game-players are good at reasoning rationally about games. Behavioural game theory studies (a) how *real* rather than *idealized* subjects actually behave when playing games, and (b) how

idealized (fully rational) subjects should behave if some of the other players depart from rationality in some way.

6. *Extensive-form games* and their solution concepts.
7. *Bargaining problems*: that class of games in which two players bargain over a certain 'pie'. There are two ways of trying to make solution-predictions for real-world bargaining problems:
    a. Try to understand the details of the problem closely, and then model the problem as an *extensive-form game*.
    b. Reason toward a solution-prediction based on certain intuitively plausible 'axioms' about properties that the solution to the problem should satisfy. It turns out that if you accept *Nash's* proposed axioms for bargaining problems, then you only to learn a few bits of information about a problem (the size of the pie; what will happen if agreement cannot be reached) in order to be able to pin down the solution predicted by the axioms. For certain sorts of problem, this *Nash solution*[1] will match the prediction that you'd get from modelling the precise details of the problem using an *extensive-form game*; for other sorts of problem, the predictions from these two approaches will differ.
8. *Evolutionary game theory*: you'll study how populations of game-players are likely to 'evolve' over time – in the narrow biological sense, but also in a broader sense, in which there is *no 'replacement'* of agents in the population over time, but in which agents change their strategies by *learning* over time which is best. As part of your study of the application of game theory to evolutionary biology, you'll learn a new solution concept for *strategic games* ('evolutionary stability').
9. *Information transmission* (reputation, signalling, etc.) in the context of *imperfect-information extensive-form games*.
10. *Theory of auctions*, and …
11. … *mechanism design*. There is some crossover between these two: the VCG (Vickrey-Clarke-Groves) principles can be used to design efficient rules for auctions, but also to design efficient mechanisms for public-goods provision, etc.
12. *Repeated games*. These are *extensive-form games* in which a particular *strategic game* (called the 'stage game') is played once, then the results are observed; then it's played again, and the results are observed; and then so on (either for a finite number of periods, or ad infinitum). There are several theorems that relate repeated games to certain solution concepts, known as 'folk theorems'.

A few pointers to bear in mind before you begin the course:

- When we study games with **imperfect information**, it's important to keep in mind that we *always* assume that *all* players are *fully informed* of the *structure of the game* (– there are devices that we can use within this structure to model the idea of imperfect information). This is a cardinal rule in the game theoretic approach to modelling imperfect information.
- Osborne's *Introduction to Game Theory* is a very good textbook, and I recommend that you study all of the relevant chapters closely, even those sections whose concepts you think you've covered in core micro. Bear in mind, however, that Osborne's treatment of

[1] No relation to the Nash equilibrium solution concept for *strategic games*.

Bayesian games and *strategic games with a correlating device* differs from the Oxford lecturer's treatment: I'd advise that you study these chapters of Osborne quickly – *if at all* – and focus instead on the lecture slides and (if required) the past exam solutions available on Weblearn.

- In my supplementary notes, I occasionally use 'function notation', like this: $f: A \rightarrow B$. In case you haven't come across this notation, it should be read as follows: 'f is a function from the domain [set] A to the codomain [set] (sometimes called 'range') B'.
- When you're revising for the final exam, it will probably be prudent to only revise a subset of topics from the course. I would personally recommend that you *steer clear of* the following topics, unless you have a particular affinity for them:
    - The more complex solution concepts for *extensive-form games with imperfect information*.
    - Mechanism design (VCG mechanisms)

  Questions on these topics are, in my opinion, often substantially trickier than those on other topics. I also personally chose not to revise *information transmission* games. If you do revise information transmission games (or, more generally, any of the topics taught by Peter Eso), my advice would be to *know the lecture slides well*: exam questions have reasonably often been based on a game discussed in the lectures (e.g. the Beer-Quiche game, or the Modified Centipede game).