

The Gale-Shapley Matching Algorithm

Harry R. Lloyd, August 1, 2019

1 The Matching Problem

Common characteristics of matching problems:

- Participants on both sides of the market may have preferences over potential matches
- Participants may lie to gain advantage
- Monetary transfers are not allowed

2 Stable Matching

Our goal is to establish stable pairings: Each α -agent should be matched to each β -agent in such a way that – no α -agent (him) would prefer a different β -agent (her) who would also prefer him over the α -agent allocated to her.

3 The Gale-Shapley Deferred Acceptance Matching Algorithm

3.1 The Algorithm

1. START: each α -agent names his favourite β -agent
2. LOOP:
 - (a) β -agents demanded by multiple α -agents *provisionally* pick an α -agent
 - (b) α -agents not picked by a β -agent choose again, with the restriction that they cannot choose a β -agent who rejected them before.
3. LOOP is repeated until each α -agent is matched with a β -agent

3.2 Properties

1. **If** every participant behaves truthfully,
then: the algorithm leads to the stable matching that is optimal for the α -agents.
2. It is a dominant strategy for the α -agents to tell the truth.
3. **If** there are multiple possible stable matchings,
then: the β -agents can manipulate the outcome by misreporting to obtain their favourite pairing.
 - Note, however, that
if there are multiple possible stable matchings,
then: there can exist *no* stable matching algorithm that will be impossible to manipulate.

3.3 Modifications

The algorithm can be modified to deal with cases where each agent starts with an endowment.