Evolutionary Game Theory for Symmetric Games: Statics

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1 The Model

- Payoffs measure reproductive fitness
- Each player is programmes to follow a certain mode of behaviour (with high probability inherited from its parents)

2 Evolutionarily Stable Strategies

2.1 Rationale

α^* is an ESS $\Leftrightarrow$ mutants will be *driven out* of a population of α^* players …

…for any sensible system of dynamics – i.e. any system of dynamics that favours high-payoff strategies

(…provided that only a small fraction of the population can mutate at once).

2.2 Definition

An evolutionarily stable strategy (ESS) in a symmetric two-player strategic game is a (possibly mixed) strategy α^* such that:

1. (α^*, α^*) is a Nash equilibrium, and
2. For every $\beta \neq \alpha^*$ that is a BR to α^*, α^* is a better response to β than β is to itself [i.e.: $U(\beta, \beta) < U(\alpha^*, \beta)$].

2.3 ESS and Strict NE

$$(\alpha^*, \alpha^*) \text{ is a strict NE} \Rightarrow \alpha^* \text{ is an ESS}$$

2.4 Finding ESS

Procedure: For each α^* such that (α^*, α^*) is a Nash equilibrium:

1. let β = the *arbitrary* mixed strategy $(p_1, p_2, \ldots)$ over the action space;
2. assume that $U(\beta, \beta) \geq U(\alpha^*, \beta)$;
3. you will either (i) isolate a set of counterexamples to α^* being an ESS, or (ii) show that $\beta = \alpha^*$, and thus that α^* *is* an ESS.