Evolutionary Game Theory for Symmetric Games: Dynamics

Harry R. Lloyd, August 1, 2019

1 Preliminaries

1.1 Mono- and Poly-morphic Populations

monomorphic population: every player has the same (pure or possibly mixed) strategy

polymorphic population: different players may have different strategies

1.2 State

The state of a process at some point in time is a list specifying the strategies being played at that point in time, and the proportion of players playing them.

1.3 Expected Payoff

The state of a process determines the expected payoff of any particular strategy.

1.4 Basin of Attraction

Where s^* is a state of some process, the basin of attraction for s^* is the set of states from which that process will converge to s^*, *absent any mutations*.

1.5 Absorbing States

A state of a process that is stable once reached is referred to as an absorbing state.[1]

[1]Note that provided the *proportions* remain constant, we *can* have change at the *individual* level.

1.6 Stochastically Stable Equilibrium

Suppose that there are two quasi-absorbing states, E_1 and E_2, whose basins of attraction partition the set of states. If the probability of moving from E_1 to E_2 is less than the probability of moving from E_2 to E_1, then E_1 is the unique stochastically stable equilibrium of the process.

(In many setups, the probability of moving from E_1 to E_2 will depend on the size of E_1's basin of attraction.)

2 Replicator Dynamics

2.1 Setup

We assume that all players play pure strategies.

2.2 The Replicator Equation

2.2.1 My Notation

$\dot{p}_S = p_S \lambda (f_S(\mathbf{p}) - \bar{f}(\mathbf{p}))$

Or equivalently: $g_{p_S} = \lambda (f_S(\mathbf{p}) - \bar{f}(\mathbf{p}))$

Thus: $g_{p_S} \propto (f_S(\mathbf{p}) - \bar{f}(\mathbf{p}))$

Key:

p_S	the proportion of players playing A in the population
$\mathbf{p}$	the present state of the population
$f_S(\mathbf{p})$	the expected fitness of a player playing strategy A, given $\mathbf{p}$
$\bar{f}(\mathbf{p})$	the average fitness of the population as a whole, given $\mathbf{p}$
λ	a scaling factor

2.2.2 Beggs' Notation

$\dot{p}_S = p_S \lambda ([Ap]_S - pAp)$

2.3 Using the Replicator Equation

In some settings (especially simple games) it can be helpful to note that $f_S(\mathbf{p}) > \bar{f}(\mathbf{p})$ iff $f_S(\mathbf{p}) > \bar{f}_{-S}(\mathbf{p})$

3 Extensions to Replicator Dynamics

Mutation can be modelled by introducing stochasticity into replicator dynamics.

We can also divide the row and column players into separate populations.

4 Best-Reply Dynamics

There is no single standard specification for the model,[2] but the general idea is that at the end of any given period, any given player will with positive probability be selected to revise their strategy, and that players so selected will choose a best response to the current state of the process.

5 Best-Reply Dynamics with Errors[3]

There is again no single standard specification for the model, but the general idea is that at the end of any given period, any given player will with positive probability be selected to revise their strategy, and that players so selected will choose a best response to the current state of the process with probability $(1-\varepsilon)$, and a non-best response with probability ε.

6 Behaviour of Stochastic Processes in the (Very) Long Run

6.1 Proving that an Absorbing State will be Reached

It is useful to note that as time $\rightarrow \infty$, the probability of even the most unlikely event happening at least once $\rightarrow 1$.

We can use this fact to our advantage when we wish to prove that an absorbing state will eventually be reached: all we need to do is to show after *some* event that will eventually be reached, an absorbing state will follow (in, say, one or two time periods).

[2]Note, for example, that the possibility of having mixed strategy equilibria will depend closely on what happens when there are multiple responses for a player to choose from: some models, for instance, incorporate sampling variation.

[3]This is *one* way of incorporating stochasticity into the best-reply dynamics framework.

6.2 When there is No Absorbing State

In the (very) long run, the behaviour of a stochastic process with no absorbing state will be independent of its present conditions.

7 Graphing Non-Stochastic Systems

When there are *N* variable parameters in the a stochastic system, we can graph the dynamics of the system using an *N* dimensional graph, with nodes to represent absorbing states, and arrows to represent dynamical changes. **Example:**