Correlated Equilibrium

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CE is an appropriate solution concept for games in which there exists or can be built a *signalling device*.

1 Information Structures

An information structure models a signalling device.

1.1 Information Structure with Common Priors

$\langle\ \Omega$	a *finite* set of outcomes of the signalling device
π	**priors:** $\pi(\omega)$ = the probability that the players attach to $\omega \in \Omega$ *prior* to receiving any signal
$\{\mathcal{P}_i\}_{i \in N}\ \rangle$	a *partition* of Ω: player i's *information partition* tells us about player i's information after the players have received their signals

2 Posterior Probabilities

Posterior probabilities are derived using Bayes' Theorem:

$$\omega \in \mathcal{P}_i(\omega') \Rightarrow \pi(\omega \,|\, \mathcal{P}_i(\omega')) = \frac{\pi(\omega)}{\pi(\mathcal{P}_i(\omega'))}$$
$$\omega \notin \mathcal{P}_i(\omega') \Rightarrow \pi(\omega \,|\, \mathcal{P}_i(\omega')) = 0$$

3 Laplacian Priors

Laplacian prior: a prior that says: 'each of my opponent's actions are equally likely'.

It is reasonable to assume Laplacian priors in cases where the players 'know nothing'.

4 Expanded Games

$$\text{expanded game} = \text{strategic game} + \text{information structure}$$

4.1 Strategies

A pure strategy for player i in $\underbrace{\langle N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle}_{\text{strategic game}} + \underbrace{\langle \Omega, \pi, \{\mathcal{P}_i\}_{i \in N} \rangle}_{\text{information structure}}$ is:

$$\begin{aligned} \text{a function} \quad & \sigma_i : \Omega \to A_i \\ \text{such that} \quad & \omega' \in \mathcal{P}_i(\omega) \Rightarrow \sigma_i(\omega) = \sigma_i(\omega') \end{aligned}$$

5 Correlated Equilibrium

5.1 Types of CE

The lecturer's preferred definition of CE:

'*objective* …	– priors are *common* rather than *heterogeneous*
… CE	s.t. the requirement of …
… *interim optimality*'	– players decide on actions after receiving their signals, rather than committing to responding to each signal in a certain way *based on their prior probability beliefs*

5.2 Definition

σ is a correlated equilibrium of $\langle N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle + \langle \Omega, \pi, \{\mathcal{P}_i\}_{i\in N}\rangle$ iff:
for all players i and information sets $\mathcal{P}_i$:
after receiving the signal $\mathcal{P}_i(\omega)$, σ_i is optimal for player i given σ_{-i}.

5.3 Relationship to Other Solution Concepts

On the lecturer's definition:

$$\textit{\textbf{NE}} \subset \textit{\textbf{CE}} \subset \text{ the set of profiles of } \textit{\textbf{rationalizable}} \text{ actions}$$

5.4 Generating a Correlated Equilibrium Payoff-Equivalent to a Particular Mixed-Nash Equilibrium

Example: 2017 finals paper, Q1(d).

Suppose that in the mixed Nash equilibrium in question, player 1 mixes $(p, 1-p)$ over (T, B), and player 2 mixes $(q, 1-q)$ over (L, R).

Method: we let Ω be 'the canonical information structure' $\{TL, TR, ML, MR\}$; we define the common prior belief π as follows:

$$\begin{aligned}
\pi(TL) &= pq \\
\pi(BL) &= (1-p)q \\
\pi(TR) &= p(1-q) \\
\pi(BR) &= (1-p)(1-q)
\end{aligned}$$

and we define the players' information partitions as follows:

$$\begin{aligned}
\mathcal{P}_1 &= \{\{TL, TR\}, \{BL, BR\}\} \\
\mathcal{P}_2 &= \{\{TL, BL\}, \{TR, BR\}\}
\end{aligned}$$

Explanation: we can gloss CE as generalising mixed NE: mixed NE is CE with *private* correlating devices. Well: the correlating device over the canonical information structure that I have just defined can be seen as 'flipping a coin for each player', with the total profile of instructions from all of those coin flips comprising the state in Ω that gets realised. Put like this, it becomes obvious that the CE strategy for apparently 'mimicking' a mixed NE that I have just described is in actual fact just *another way of expressing* the original mixed NE.