# *Correlated Equilibrium*

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CE is an appropriate solution concept for games in which there exists or can be built a *signalling device*.

# 1 Information Structures

An information structure models a signalling device.

## 1.1 Information Structure with Common Priors

| | |
|---|---|
| $\langle\ \Omega$ | a *finite* set of outcomes of the signalling device |
| $\pi$ | **priors:** $\pi(\omega)$ = the probability that the players attach to $\omega \in \Omega$ *prior* to receiving any signal |
| $\{\mathcal{P}_i\}_{i \in N}\ \rangle$ | a *partition* of $\Omega$: player $i$'s *information partition*<br>player $i$'s information after the players have received their signals |

# 2 Posterior Probabilities

Posterior probabilities are derived using Bayes' Theorem:

$$\omega \in \mathcal{P}_i \Rightarrow \pi(\omega \mid \mathcal{P}_i) = \frac{\pi(\omega)}{\pi(\mathcal{P}_i)}$$
$$\omega \notin \mathcal{P}_i \Rightarrow \pi(\omega \mid \mathcal{P}_i) = 0$$

# 3 Laplacian Priors

Laplacian prior: a prior that says: 'each of my opponent's actions are equally likely'.

It is reasonable to assume Laplacian priors in cases where the players 'know nothing'.

# 4 Expanded Games

$$\text{expanded game} = \text{strategic game} + \text{information structure}$$

## 4.1 Strategies

A pure strategy for player $i$ in $\underbrace{\langle N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle}_{\text{strategic game}} + \underbrace{\langle \Omega, \pi, \{\mathcal{P}_i\}_{i\in N}\rangle}_{\text{information structure}}$ is:

$$\begin{aligned} \text{a function} \quad & \sigma_i : \Omega \to A_i \\ \text{such that} \quad & \omega' \in \mathcal{P}_i(\omega) \Rightarrow \sigma_i(\omega) = \sigma_i(\omega') \end{aligned}$$

# 5 Correlated Equilibrium

## 5.1 Types of CE

The lecturer's preferred definition of CE:

| | |
|---|---|
| '*objective* … | – priors are *common* rather than *heterogeneous* |
| …CE | s.t. the requirement of … |
| … *interim optimality*' | – players decide on actions after receiving their signals, rather than committing to responding to each signal in a certain way *based on their prior probability beliefs* |

## 5.2 Definition

$\sigma$ is a correlated equilibrium of $\langle N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle + \langle \Omega, \pi, \{\mathcal{P}_i\}_{i \in N} \rangle$ iff:
for all players $i$ and information sets $\mathcal{P}_i$:
after receiving the signal $\mathcal{P}_i$, $\sigma_i$ is optimal for player $i$ given $\sigma_{-i}$.

## 5.3 Relationship to Other Solution Concepts

On the lecturer's definition:

$$\boldsymbol{NE} \subset \boldsymbol{CE} \subset \text{the set of profiles of } \textbf{\textit{rationalizable}} \text{ actions}$$

## 5.4 Generating a Correlated Equilibrium Payoff-Equivalent to a Particular Mixed-Nash Equilibrium

**Example:** 2017 finals paper, Q1(d).

Suppose that in the mixed Nash equilibrium in question, player 1 mixes $(p, 1-p)$ over $(T, B)$, and player 2 mixes $(q, 1-q)$ over $(L, R)$.

Method: we let $\Omega$ be 'the canonical information structure' $\{TL, TR, ML, MR\}$; we define the common prior belief $\pi$ as follows:

$$\begin{aligned}
\pi(TL) &= pq \\
\pi(BL) &= (1-p)q \\
\pi(TR) &= p(1-q) \\
\pi(BR) &= (1-p)(1-q)
\end{aligned}$$

and we define the players' information partitions as follows:

$$\begin{aligned}
\mathcal{P}_1 &= \{\{TL, TR\}, \{BL, BR\}\} \\
\mathcal{P}_2 &= \{\{TL, BL\}, \{TR, BR\}\}
\end{aligned}$$