

BAR GAINING

• Bargaining problem: $\langle U, d \rangle$

• Assumptions: ① $d \in U$

② $\exists (v_1, v_2) \in U$ s.t. $(v_1, v_2) \succ d$

③ U is convex

④ U is compact

• Bargaining solution: $F: \{\langle U, d \rangle\} \rightarrow U$

• Axioms:

(WP) $\nexists (v_1, v_2) \in U$ s.t. $(v_1, v_2) \succ F\langle U, d \rangle$

(SYM) IF: $(v_1, v_2) \in U$ iff $(v_2, v_1) \in U$

and $d_1 = d_2$

THEN: $F_2\langle U, d \rangle = F_1\langle U, d \rangle$

(INV) IF: ~~U'~~ $\alpha_1, \alpha_2 \in \mathbb{R}_{>0}, \beta_1, \beta_2 \in \mathbb{R}$

and $U' := \{(\alpha_1 u_1 + \beta_1, \alpha_2 u_2 + \beta_2) : (u_1, u_2) \in U\}$

and $d' := (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$

THEN: $F\langle U', d' \rangle$

$= (\alpha_1 F_1\langle U, d \rangle + \beta_1, \alpha_2 F_2\langle U, d \rangle + \beta_2)$

(IIA) IF: $U' \subseteq U$

and $F\langle U, d \rangle \in U'$

THEN: $F\langle U', d \rangle = F\langle U, d \rangle$

- Nash's bargaining theorem:

F satisfies WP, SYM, INV and IIA, iff:

$$F := \arg \max_{(v_1, v_2)} \underbrace{(v_1 - d_1)(v_2 - d_2)}_{\text{rectangular hyperbola}}$$

$$\text{s.t. } (v_1, v_2) \in U$$

$$\text{and } (v_1, v_2) \geq d$$

- Results:

↳ the Nash bargaining solution is the same as the limit of the SPE outcome of an extensive-form game of probabilistic offers with a fixed probability α of breakdown after each refusal, as $\alpha \rightarrow 0$ (where d is the pair of breakdown payoffs).

↳ under the Nash bargaining solution, the player with less risk-aversion gets more of the pie.

- An extensive-form game is stationary if subgames alternate between a certain finite set of types.

- A strategy is stationary if each player always makes the same proposals and always accepts the same set of proposals.